Transonic Region of a Supersonic Boundary Layer Turning a Sharp Corner

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Theme

THIS paper presents solutions for the flow variables in the transonic region of a steady, laminar, supersonic boundary layer turning a corner. The analysis holds in those cases where the corner is sharp and the pressure drop across the corner is large enough that separation occurs, viscous effects in the corner region are important only in a negligibly thin sublayer, and the base pressure to which the flow expands is less than the sonic pressure, P^*, corresponding to the flow conditions at the wall upstream of the corner. It has been shown[1,2] that under these conditions, inviscid flow equations may be employed in a region extending a few boundary-layer thicknesses upstream of the corner; the sonic line intersects the corner to the scale of the inviscid region. Since this analysis is concerned only with the transonic flow in the inviscid region, it is valid in a region with dimensions small compared to those associated with the inviscid region, yet large compared to those associated with the viscous sublayer. It supplements Refs. 1 and 2 by supplying detailed transonic solutions not given in these references. Hence, the solutions given here fulfill the same role in the problem of the turning of a supersonic boundary layer around a corner, that Vaglio-Laurin's[3] transonic corner flow solutions do in the problem of an inviscid flow turning a corner. Physical applications are the corner flows occurring at the lip of a rocket nozzle, at a rearward facing step, or at the base corner of a re-entry vehicle, for example.

A systematic method of treating rotational transonic inviscid flow around a convex corner was presented in a previous paper by the present authors.[4] In that paper, the method was applied to the transonic region in the case where the boundary layer approaching the corner had no pressure gradient or heat transfer. In this paper, the method is extended to cover the case of a laminar supersonic boundary layer with pressure gradient and heat transfer; it is demonstrated that the shape of the sonic line and the pressure gradient are fundamentally changed from the simple case.

Contents

The coordinate system and the region under consideration are illustrated in Fig. 1. (Bars denote dimensional quantities.) The general method employed is described in Ref. 4. The equations for two dimensional inviscid rotational flow are employed; the boundary conditions are simply that upstream of the corner the flow is parallel to the wall and that in the limit as the corner is approached along a radial line from downstream of the corner, the solutions tend to the Prandtl-Meyer solutions. Solutions for

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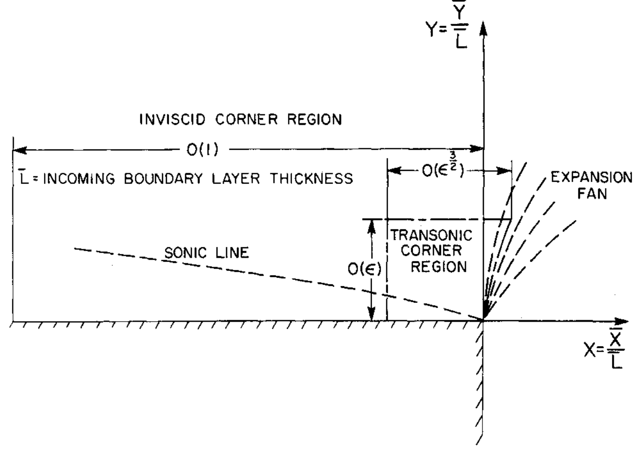

Fig. 1 Sketch of corner showing flow structure, order of inviscid and transonic regions, and coodinate system.

the velocity components are found in terms of asymptotic expansions about sonic velocity. The boundary-layer aspects of the flow are introduced into this inviscid problem by taking the entropy and stagnation enthalpy gradients to be those associated with the desired boundary-layer flow. The solution involves the use of similarity solutions and matched asymptotic expansions.

In this paper, solutions are presented for the transonic region upstream of the first left-running characteristic of the expansion fan (Fig. 1). This region, in turn, can be divided into two regions, that between this left-running characteristic and the last right-running characteristic to intersect the wall (at the corner), and the region upstream of the last right-running characteristic (Fig. 2). The solutions in the latter region are particularly simple and important, this region including the sonic line and the wall, and

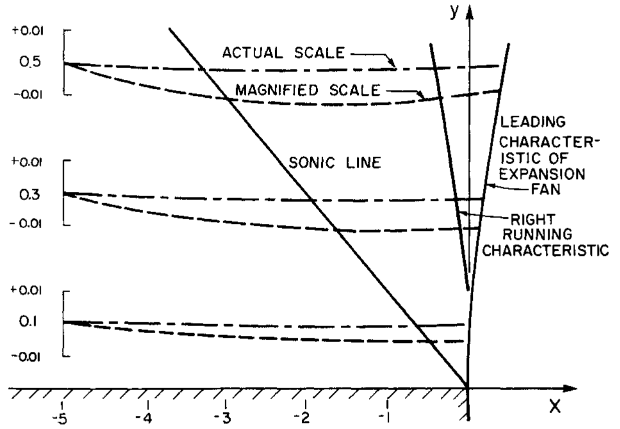

Fig. 2 Structure of flowfield near corner from numerical solution for $\gamma = 1.4$, $\varepsilon = 0.1$, $K = 1$, $C_0 = \frac{9}{4}$, and $C_{\frac{1}{2}} = 1$; - — - streamlines to scale of coordinate system; — — — streamlines with exaggerated vertical scale to illustrate variations.

are presented here as an example of the solutions found. Thus, upstream of the right-running characteristic,

$$U = 1 + C_o Y + KX + \tfrac{2}{3} C_{1/2} Y^{3/2} + \ldots \quad (1)$$

$$V = (\gamma+1)^{1/2} [\tfrac{1}{2} C_o K Y^2 + K^2 XY + \ldots] \quad (2)$$

where U and V, the x and v velocity components, respectively, are dimensionless with respect to the sonic velocity, and X and Y are dimensionless with respect to the boundary-layer thickness at the corner, represented in Fig. 1, by L. K is a constant to be found by matching with the actual outer flow in which the transonic region is embedded, and γ is the ratio of specific heats. C_o and $C_{1/2}$ are constants which depend on the shear stress and heat transfer at the wall, the pressure gradient, and the wall temperature, all evaluated at the corner. Of course, only two of these four quantities may be specified independently.

From Eq. (1), the sonic line ($U = 1$) is given by

$$\tfrac{2}{3} C_{1/2} Y^{3/2} + C_o Y = K(-X) \quad (3)$$

Thus, the sonic line shape depends, through C_o and $C_{1/2}$, on the pressure gradient and heat transfer at the wall, and is to first approximation, a straight line. Also, since the pressure perturbation is proportional to the velocity perturbation, it is seen from Eq. (1), that the pressure gradient is a constant in the transonic region.

The solutions are valid in the transonic region indicated in Fig. 1. That is, the order of X and Y are limited to

$$X = O(\varepsilon^{3/2}) \quad (4)$$

$$Y = O(\varepsilon) \quad (5)$$

where $\varepsilon = O(U-1)$ is the order of the deviation of the flow from sonic velocity. Typically, $\varepsilon \leqq 0.1$.

Numerical solutions were performed for typical values of the parameters and are shown in Fig. 2. Because the actual streamline variation is so small, to the scale shown, an exaggerated scale is also employed to illustrate the streamline form. The sonic line is seen to approach the corner tangent to a straight line which intersects the corner at an acute angle relative to the upstream wall. The magnitude of the angle depends on the boundary-layer pressure gradient and heat transfer at the corner.

References

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